AT55
Schwind

V606 771
066228

National Aeronautics and Space Administration
Goddard Space Flight Center, Greenbelt, Md.
(NASA Contract No. NAS-5-2078)

13p

ST - AD - 10 089

NASA TT F-8653

X64 12712

Code 2D

N 71-71213
(ACCESSION NUMBER)
13
(PAGES)
(NASA CR OR TMX OR AD NUMBER)
(THRU)
NONE
(CODE)
(CATEGORY)
FACILITY FORM 602

MASS SELECTION IN TUBES MOVING IN A RAREFIED GAS

[Russian title underlined on next page]

by

A. I. Repnev
[USSR]

18 Dec. 1963 13 p refs

Transl. into ENGLISH by Consultants and Designers, Inc. from Tr. Tsentr. Aerolog. Observ. (Moscow), no. 29, 1960 p 66-73

Unc.
"Available to U.S. Government Agencies and U. S. Government Contractors Only"

CONSULTANTS AND DESIGNERS, INC.
Arlington, Virginia
December, 1963

NASA TT F-8653

DRAFT TRANSLATION

ST - AD - 10 089

DEC 18 1963

MASS SELECTION IN TUBES MOVING IN RAREFIED GAS*

by A. I. REPNEV

Trudy Tsentr.Aerol.Inst.
No. 29, pp 66 – 73
Gidrometeoizdat, Moskva 1960.

A B S T R A C T

During mass-spectrometric investigations of the gravitational separation in the atmosphere at great altitudes by way of measurements of argon to nitrogen concentration ratios, and also during the investigations of the atmosphere in general, the accounting of the mass selection phenomenon has a great significance.**

The mass selection sets in at apparatus' interaction with a gas mixture in thermal equilibrium and rarefied to the extent that particles of each of the components. flow independently only in relation to their own mass (i. e. in free molecular flow).

The mass selection appears in a free molecular flow at nonstationary processes and in the presence of density gradients in the apparatus.

This papers considers the flow of gas in the tube for its various positions relative to incident flow. From the material expounded, the conclusion is derived, that when the incident flow hits the aperture the content in heavy gases in the apparatus is increased. When the directed flow is such, that it does not reach the apparatus, i. e. in case of Knudsen flow, the inflow results in the increase of light gas content in the apparatus. The current results are compared with experiments carried out in the U.S.A., and with later experiments during which the essay vessel was connected with the atmosphere by long tube

* MASSOVAYA SELEKTSIYA V TRUBKAKH DVIZHUSHCHIKHSYA V RAZREZHENNOM GAZE.

** We shall not consider here any questions linked with particle absorption in the walls or at chemical reactions.

"Available to U.S. Government Agencies and U.S. Government Contractors Only"

13712

oriented nearly along the flow. Increase in light rather than heavy gases was then obtained, contrary to theory. However, control measurements gave contradictory results. But in any case, no strictly free-molecular flow apparently took place. AUTHOR

* * *

To start, we shall examine the process of apparatus' filling with gases of the surrounding atmosphere.

Assume that at the time $t = 0$, the discharged device, placed in a body moving with a velocity V, opens up in the atmosphere. The apparatus is an enclosed volume with an aperture. The density of particles in 1 cm^3 of the atmosphere is N, the most probable velocity of particles having a Maxwellian distribution is

$$v=\sqrt{\frac{2RT}{M}},$$

where T is the temperature of the medium, R is the universal gas constant, M — the molecular weight of the gas. During the time dt

$$\frac{Nv}{2\sqrt{\pi}}\chi(\beta)S\,dt$$

particles enter the volume W through the aperture of surface S, with the result that the density of particles in the volume increases by

$$\frac{Nv}{2\sqrt{\pi}}\chi(\beta)\,\frac{S}{W}\,dt.$$

Here

$$\chi(\beta)=e^{-\beta^2}+\sqrt{\pi}\beta\,[1+\Phi(\beta)], \qquad (1)$$

$$\beta=\frac{V}{v}\sin\Theta,$$

$$\Phi(\beta)=\frac{2}{\sqrt{\pi}}\int_0^{\beta}e^{-s^2}ds,$$

where Θ is the angle between the velocity vector and aperture plane [1]. Upon hitting the wall, part of the particles flying inside the volume, will take a different temperature. We shall estimate for the sake of

simplicity, that the particles are fully accomodated. Then a flux of particles, adapted to wall temperature T_1, will outflow from the manometer volume. This will lead to a decrease of particle density N_1 by

$$\frac{N_1 v_1}{2\sqrt{\pi}} \frac{S}{W} dt,$$

in the volume. Here

$$v_1 = \sqrt{\frac{2RT_1}{M}},$$

and N_1 is the density of particles with a temperature T_1.

We shall estimate that N depends on time, for the body moves upward with the vertical component of the velocity V_t in the atmosphere where pressure dropa with the altitude according to the barometric formula. We shall admit that v and $\chi(\beta)$ are not dependent on time. Writing the particle balance equation, we shall obtain its solution in the form:

$$N_1(t) = e^{-\frac{t}{\tau_1}} \left[N_0 \chi(\beta) \frac{\tau_2}{\tau_0} \left(1 - e^{-\frac{t}{\tau_2}}\right)\right], \qquad (2)$$

Here

$$\tau_0 = \frac{2\sqrt{\pi} W}{v S}; \quad \tau_1 = \frac{2\sqrt{\pi} W}{v_1 S}; \quad \tau_2 = \frac{1}{\frac{Mg V_e}{RT} - \frac{1}{\tau_1}}.$$

N_0 denotes the density of atmosphere particles at the initial level of apparatus' opening, g is the gravitation acceleration.

The estimate shows that $\frac{1}{\tau_1} \gg \frac{Mg V_z}{RT}$ at $V_z = 1$ km/sec; $W = 1000$ cm^3; $S = 3$ cm^2; $T = 1000^\circ$ K and $T_1 = 300^\circ$ K, so that $\tau_2 \cong -\tau_1$ and

$$N_1(t) = N_0 \chi(\beta) \frac{v}{v_1} \left(1 - e^{-\frac{t}{\tau_1}}\right). \qquad (3)$$

The particle concentration ratio of two gases in the atmosphere is expressed by their ratio in the apparatus' volume, and has the form:

$$\frac{N_0}{N_0^*} = \frac{N_1}{N_1^*} \frac{\chi^*(\beta^*)}{\chi(\beta)} \frac{\left(1 - e^{-\frac{v_1^* St}{2\sqrt{\pi} W}}\right)}{\left(1 - e^{-\frac{v_1 St}{2\sqrt{\pi} W}}\right)} = \frac{N_1}{N_1^*} \mu_t, \qquad (4)$$

since in a free molecula flow the gases of the mixture behave independently (the sign * distinguishes two gases with molecular weights M and M*, β depends on M through v).

The factor μ_t, by which the measured gas ratios must be multiplied in order to obtain their ratio, shall be called "mass selection factor".

Under equilibrium conditions, when the temporal terms cease to play any role, the selection is determined by the relation

$$\mu_0 \cong \frac{\chi^*(\beta^*)}{\chi(\beta)}. \qquad (5)$$

If the body motion velocity is greater than the mean thermal velocity of particles in the atmosphere (in practice it is sufficient for the condition to be fulfilled that $\beta > 1.5 \div 2$), the aperture of the apparatus captures all the particles in front of it, without distinction of mass, while their outflow velocity from the volume is inversely proportional to the radical from the molecular weight. Therefore (5) passes to

$$\mu_{00} \cong \sqrt{\frac{M^*}{M}}, \qquad (6)$$

since for $\beta > 1,5 \div 2.$ $\qquad \chi(\beta) \cong 2\sqrt{\pi}\beta$

If we have measured the ratio of heavy to light gas content (M M*) the correlation (6) shows that the heavy gas content in the apparatus is increased, and in order to bring it into correspondence with its content in an undisturbed atmsophere, of interest to us, we must multiply the measured correlation by $\mu_{00} < 1$.

The time $\tau_1 \sim 10^{-2}$ sec, if $S = 3\,cm^2$, $W = 10^3 \cdot cm^3$, For $t < \tau_1$, $\mu > \mu_0$ and approaches μ_0.

Similarly, for a bottom vacuum region, or aerodynamic shadow, we shall obtain :

../..

$$\mu_1' = \frac{\chi'^*(\beta^*)}{\chi'(\beta)} \frac{\left(1 - e^{-\frac{v_1^* St}{2\sqrt{\pi} W}}\right)}{\left(1 - e^{-\frac{v_1 St}{2\sqrt{\pi} W}}\right)}, \tag{7}$$

where

$$\chi'(\beta) = e^{-\beta^2} - \sqrt{\pi}\beta[1 - \Phi(\beta)]. \tag{8}$$

For

$$t > \tau_1$$
$$\mu_0' \cong \frac{\chi'^*(\beta^*)}{\chi'(\beta)}. \tag{9}$$

If moreover $\beta > 1.5 \div 2$, we have

$$\mu_{00}' \cong \frac{M}{M^*} e^{-\beta^2\left(\frac{M}{M^*} - 1\right)} \tag{10}$$

and
$$\mu_0' \to \infty \quad \text{at} \quad \beta \to \infty, \quad \text{if} \quad M > M^*$$
$$\mu_0' \to 0 \quad \text{at} \quad \beta \to \infty, \quad \text{if} \quad M < M^*.$$

For $t < \tau_1$, $\mu' < \mu_0'$ and approaches μ_0

The inleakage under the effect of the incident flow will be yet examined in the stationary case of a tube oriented along the flow. As is shown in [2], the density of particles then increases as the origin of the tube gets farther, according to the law:

$$N_{10}(x) = \frac{N}{2}\left\{\sqrt{\frac{T}{T_1}}\chi(\beta)[2 - \varphi(x)] + \varphi(x)[1 + \Phi(\beta)]\right\}, \tag{11}$$

where $N_{10}(x)$ is the aggregate density of particles with temperatures T and T_1.

In this particular case $\varphi(x)$ is approximated by the following expression:

$$\varphi(b) = 1 - \sqrt{\pi}\, b\, e^{b^2}[1 - \Phi(b)], \tag{12}$$

Where

$b = \sqrt{\pi}\alpha \frac{\gamma(\beta_0)^2}{\chi(\beta_0)} \frac{x}{r}$, $\beta_0 = \frac{V}{v}$, r is the radius of the tube, α is the share of folly accomodating particles (it is assumed that $1 - \alpha$ is specularly reflected). The ratio of two gases' concentrations in the atmosphere, expressed through their ratio measured at a distance x from the inlet is determined by the formula

...

$$\frac{N}{N^*} = \frac{N_{10}}{N_{10}^*}\mu, \qquad (13)$$

but μ will have the form:

$$\mu = \frac{\sqrt{\frac{T}{T_1}}\chi^*(\beta_0^*)[2-\varphi^*(b^*)]+\varphi^*(b^*)[1+\Phi(\beta_0^*)]}{\sqrt{\frac{T}{T_1}}\chi(\beta_0)[2-\varphi(b)]+\varphi(b)[1+\Phi(\beta_0)]}, \qquad (14)$$

where

$$\beta_0^* = \beta\sqrt{\frac{M^*}{M}}; \quad b^* = b\,\frac{\chi(\beta_0)}{\chi^*(\beta_0^*)}\,\frac{\alpha^*}{\alpha}\,\frac{\nu^*(\beta_0^*)}{\nu(\beta_0)}.$$

As may be seen, μ depends on the motion velocity of the body, of gas temperatures and on the distance from the inlet aperture.

In case $\beta > 1.5 \div 2$, the expression for μ is somewhat simplified:

$$\mu = \frac{2\sqrt{\pi}\sqrt{\frac{T}{T_1}}\beta_0^* + \varphi^*(b^*)\left[1-\sqrt{\pi}\sqrt{\frac{T}{T_1}}\beta_0^*\right]}{2\sqrt{\pi}\sqrt{\frac{T}{T_1}}\beta_0 + \varphi(b)\left[1-\sqrt{\pi}\sqrt{\frac{T}{T_1}}\beta_0\right]}. \qquad (15)$$

At great remoteness from the end of the tube $\varphi = \varphi^* \cong 0$, and we arrive at formulas (5) and (6), where the spatial gradient is not taken into account. Correlations (5) and (6) have the greatest practical value, for in many cases the condition can be assured, when the particle flux of the outer medium does not penetrate into the place of disposition of apparatus' receiving part ($\varphi = 0$), though it still hits its aperture. This condition will take place if the axis of the tube is significantly deflected from the direction of the flow.

In the bottom vacuum region function $\varphi(x)$ lacks the simple form (12), and therefore we limit ourselves by the region where the density gradients are absent, for which we shall have formulas (9) and (10).

It is interesting to note, that even in a quiescent apparatus $\mu \neq 1$ even if $\alpha \neq \alpha^*$.

The numerical computations of mass selection factors for certain atmosphere gases as a function of velocity and distance along the tube's axis are plotted in Figures 1 to 4. We compiled in Table 1 the values of μ_{00}, (i.e. in the region of density gradients' absence at $\beta > 1.5 \div 2$) for certain gases of the atmosphere taken relative to nitrogen.

Table 1 allows to visualize the maximum value of the correction for the mass selection.

μ_0 is plotted as a function of velocity in Fig. 1. $\mu_0 \simeq \mu_{00}$ * already for $\beta > 1.5 \div 2$, $\varphi(x)$ can be considered as a negligible value ** for heavy gases at $\frac{x}{r} \sim 20 \div 50$. This is seen from Figs. 2, 3, as well as from the estimation formula

$$\frac{x_0}{r} \geqslant 6.3\beta_0 \qquad (16)$$

which gives the ratio of tube's aperture remoteness to its radius as a function of the motion velocity, which is easily obtained from the condition that $\varphi(x_0) \ll 1$ (for example, $\varphi(x_0) \leqslant 0{,}05$). It is assumed then that at the same time $\beta > 1.5 \div 2$, while $\frac{x_0}{r}$ is such that $b \geqslant 3{,}1$ — $\Phi(b)$ is expanded in series [3], where the first term is preserved.

It may be seen from Fig. 3 that for a given velocity, μ is so much the nearer the value determined by correlation (5), that x is greater. And with the increase in velocity, the gradient region penetrates into the tube to a greater depth. The "dip" in the curves of μ dependence on $\frac{x}{r}$ then broadens, becoming progressively "shallower", merging with the line at limits.

T A B L E 1

GAS (X)	Molecular weight (g / mole)	$\mu_{00} = \sqrt{\frac{M_{N_2}}{M_x}}$
N_2	28.016	1
H	1.008	5.272
H_2	2.016	3.742
He	4.003	2.464
N	14.008	1.414
O	16.009	1.323
OH	17.008	1.283
H_2O	18.016	1.247
Ne	20.183	1.178
NO	30.008	0.9662
O_2	32.000	0.9357
Ar	39.944	0.8375
CO_2	44.010	0.7979
N_2O	44.016	0.7978
NO_2	46.008	0.7803

As the motion velocity increases, the heavy gas content drops catastrophically, for heavy gas particles have a low velocity and cannot catch up with the fast-moving body. The corrective factor such as μ', which aims at bringing the ratio of gases to that existing in the atmosphere, rises as rapidly in connection with the above for the ratio of heavy to light gas content, and approaches zero for the ratio of light to heavy gas content (see Fig. 4).

* except for hydrogen. ** $\frac{x}{r} \sim 40 \div 50$ for hydrogen.

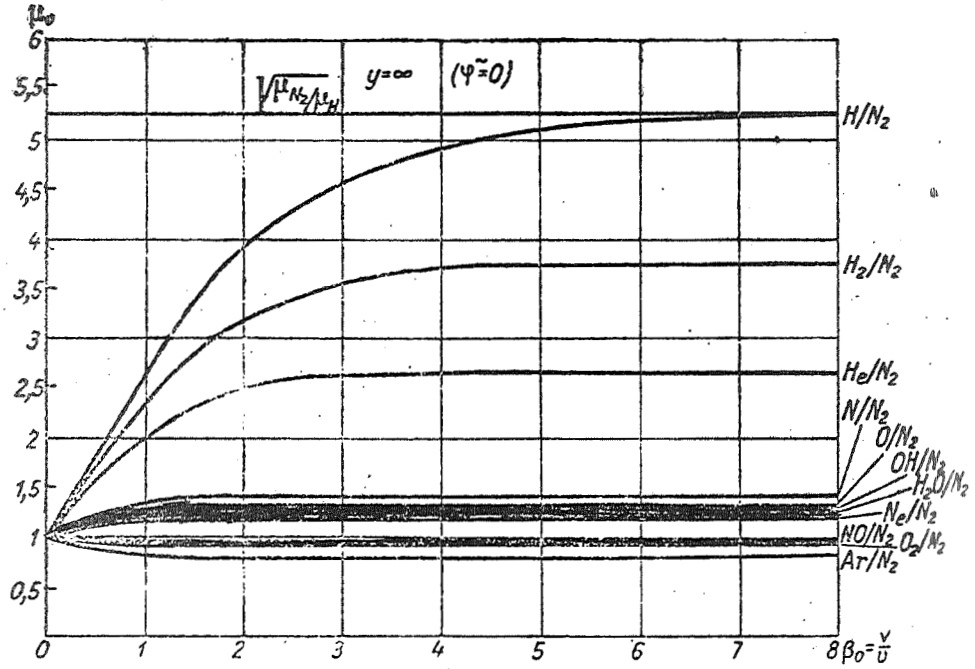

Fig. 1.- Mass selection factor μ_0 [formula (5)] in the stationay case and outside the gradient region for the ratio of various gases to nitrogen contents as a function of motion velocity.

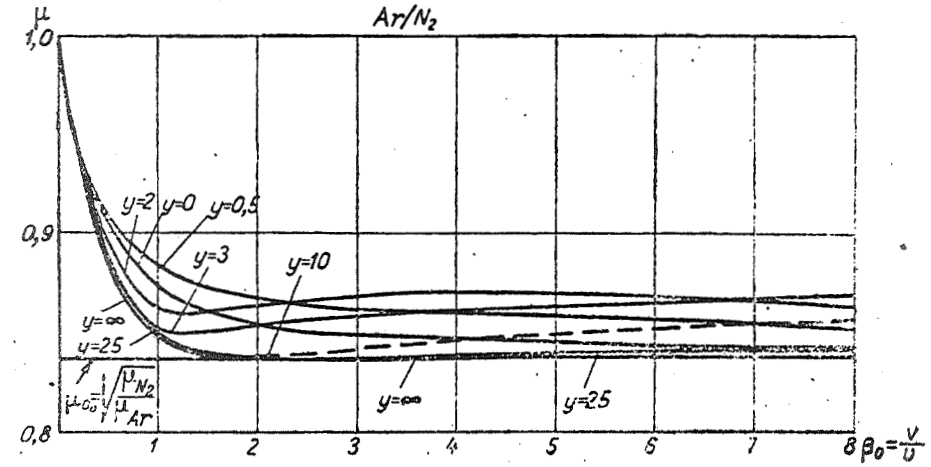

Fig. 2.- Mass selection factor μ [formula (14)] for the ratio Ar/N_2 as a function of motion velocity. It is assumed that

$$\alpha=1, \sqrt{\frac{T}{T_1}}=2.$$

Attention is drawn to the fact, that enrichment of the mixture in the apparatus with heavy gases took place at latter's filling by incident flow in all cases considered. The only exception is the bottom vacuum region.

A different picture is obtained at such filling of apparatus' volume, when the incident flow misses the aperture (velocity vector perpendicular to the normal of aperture plane). Integrating by vessel's volume the continuity equation and passing to the integral along the surface, it is easy to obtain the linear equation for finding the time variation in the apparatus' volume of gas particles' concentration with the given mass of molecules, by utilizing the Knudsen formulas [4, 5]. The solution will have a form analogus to (3):

$$N_1(t) = N_0\left(1 - e^{-\frac{t}{\tau}}\right) \cdot \frac{v}{v_1}, \qquad (17)$$

where

$$\tau = \frac{3\,x\,W}{4\sqrt{2\pi r^3 v_1}}$$

for the given tube and

$$\tau = \frac{2\sqrt{\pi}\,W}{v_1 S}$$

for the diaphragm.

Fot $t > \tau$, it is evident that $\mu \cong 1$. At $t < \tau$,

$$\mu_t \sim \sqrt{\frac{M}{M^*}} > 1 \qquad (18)$$

(provided the ratio of heavy to light gas content in the atmosphere is expressed by their ratio in the apparatus' volume), In other words, the light gas content in the apparatus is increased.

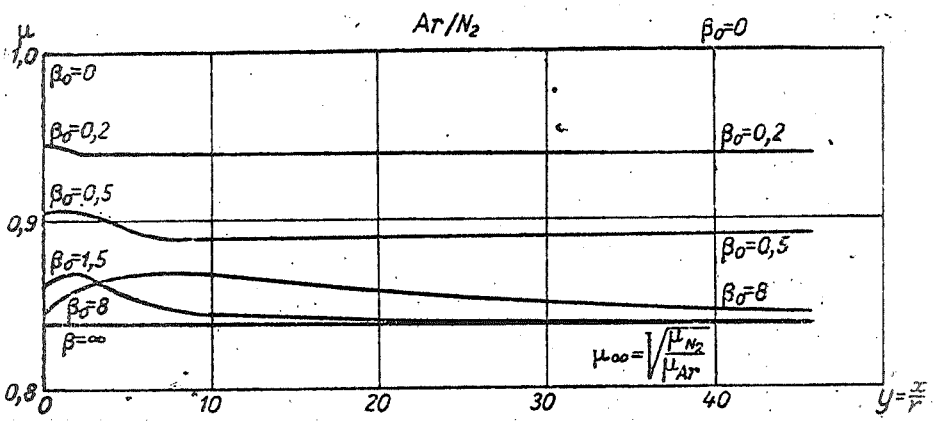

Fig. 3. Mass selection factor μ [formula (14)] for the ratio Ar/N_2 as a function of remoteness $y = \frac{x}{r}$ from the tube origin, the latter being oriented along the flux $\left(\beta_0 = \frac{V}{v}\right)$. It is assumed that $\alpha = 1$, $\sqrt{\frac{T}{T_1}} = 2$.

In conclusion we shall consider a case of gas mixture flow through a long tube, at whose ends a constant pressure drop $\frac{p - p'}{x}$ is maintained, for example by means of pumping out (as previosuly, the incident flow misses the aperture). The quantity of gas of each kind, flowing through, is determined by the Knudsen formula [4]:

$$Q = S\rho\, U = \frac{4}{3}\sqrt{2\pi}\sqrt{\frac{\rho_0}{p_0}}\, r^3\, \frac{p - p_1}{x}, \qquad (19)$$

where ρ_0 is the desnity of the gas in the tube, p_0 is its mean pressure, U — the flow velocity.

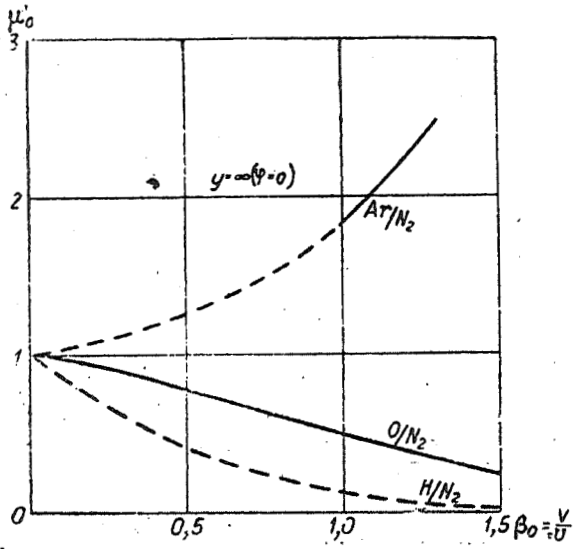

Fig. 4. - Mass selection factor in the bottom vacuum μ'_0 [formula (9)] in the stationary case outside the region of gradients for the gases' contant ratios Ar/N_2, O/N_2 and H/N_2 as a function of motion velocity.

From (19) we may express p by p_1 and introduce the mass selection factor in the sense indicated above. If we choose for Q = const the value at the end of the tube, say $S\rho_1 U_1$, we shall have:

$$\frac{N}{N^*} = \frac{N_1}{N_1^*}\,\mu = \frac{N_1}{N_1^*}\,\frac{1 + \frac{3}{4}\sqrt{\frac{\pi}{2}}\,\frac{x}{r}\,\frac{U_1\sqrt{M}}{\sqrt{RT}}}{1 + \frac{3}{4}\sqrt{\frac{\pi}{2}}\,\frac{x}{r}\,\frac{U_1^*\sqrt{M^*}}{\sqrt{RT}}}. \qquad (20)$$

Since U apparently does not depend on M, we have $\mu > 1$ (for $M > M^*$), i.e. after flow through the long tube in free-molecular flow conditions, the gas is enriched with light components. At great x, $\mu \to \sqrt{\frac{M}{M^*}}$.

It follows from the above-expounded material, that when incident flow hits the aperture, the content in heavy gases in the apparatus is increased. When the directed flow misses the aperture (we shall designate this case as Knudsen flow), at its inflow the content in light gases increases.

In principle, the distinction of the Knudsen flow from that with incident flow consists in that in the first cases there takes place a flow of equiponderant (equilibrium) gas, while in the case of incident flow there are flows of two types of particles (differing by temperatures), each of which is unequilibrated in the sense, that an unequal number of particles of that kind flow in opposite directions [2].

In the U.S.A. experiments [6], the time constant was greater than that of sampling. At the same time, in earlier experiments, the inflow was realized from a special chamber, open at the side of the incident flow. The content in heavy gases was increased in it. As to the further inflow from it, it took place in such a way, that the contemt in light gases must have increased. Therefore, a complex superimposition of two opposite effects was obtained, whose numerical accounting is difficult. Apparently, the aggregate mass effect is determined by the product μ_0 (5) and μ (20).

In later experiments, when the test vessel was connected with the atmosphere by a long tube, oriented about along the flow, the increase in light gas content was obtained, and not in heavy gases, as this follows from theory. But control measurements gave quite contradictory results. In any case, no strictly free-molecular flow apparently had taken place.

**** THE END ****

REFERENCES

[1].- E. SANGER.- Schweiz. Archiv. Angew. Wiss. Techm., 16, No. 2, 1950.

[2].- A. I. IVANOVSKIY, A. I. REPNEV.- (see current issue) , 1960.

[3].- A. N. TIKHONOV, A. A. SAMARSKIY.- Uravneniya matematicheskoy fiziki (Phys.Mathem.Equations).- M., 1950.

[4].- M. KNUDSEN. - Annalen d. Physik, 28, f. 4, 1909.

[5].- GROSHKOVSKIY.- Tekhnologiya vysokogo vakuuma. (High Vacuum Technology). -M, 1957.

[6].- R. L. BOYD, M. SEATON. - Dzh. red. Raketnyye issledovaniya verhney atmosfery (Rocket Investigations of the Upper Atmosphere). - M., 1957.

Translated by ANDRE L. BRICHANT
under Contract No. NAS-5-2078
on 16 December 1963.

DISTRIBUTION

GODDARD SPACE F.C.

610	MEREDITH	
611	McDONALD	(3)
612	HEPPNER	
614	LINDSAY	
613	KUPPERIAN	
	E. REED	(3)
615	BOURDEAU	(3)
640	HESS	(3)
651	SPENCER	
652.	TOWNSEND M.	
651	BRINTON	
	C.R.SMITH	
	H. TAYLOR	
252	WEAVER LIB [5]	

NASA HEADQUARTERS

SS	NEWELL, CLARK
SP	STANSELL
SG	NAUGLE
	FELLOWS
	DUBIN
	HIPSHER, HOROWITZ
	SCHARDT
	OPP
RR	KURZWEG
RRP	GESSOW
RV-1	PEARSON
RTR	NEILL
AFSS	SCHWIND
A-4	

OTHERS

AMES	[2]
LANGLEY	[2]
LEWIS	[2]
MARSHALL	[2]
JPL	[2]
AID	